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For hydrographic surveys conducted by the Naval Oceanographic Office, the position of the sounding vessel is determined by applying the method of adjustment by variation of parameters. Three types of navigational aids are used: ranging (ARGO, RAYDIST, LORAN-C), azimuthal (Coast Artillary Azimuth Instrument), and hyperbolic (ARGO, DECCA, HI-FIX6, LORAN-C). Given data from any combination of at least two navigational aids, a fix may be obtained using an iterative method, which applies successive adjustments to an approximate location and forces it to converge to the most probable position. The magnitude and direction of each adjustment is determined from a least squares solution that minimizes the residual differences between actual navigational observations and imaginary observations calculated as if the ship were at the approximate location.

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Sounding Vessel Position from Adjustment by Variation of Parameters

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Sponsored by:
Defense Mapping Agency
Washington, D.C.

Foreword

This report is the first of a series that will provide documentation for software being developed specifically for Naval Oceanographic Office hydrographic survey applications. New hardware, software, and procedures that maximize automation to improve efficiency while maintaining high quality of hydrographic products are being treated collectively as a Hydrographic Information Handling system. Future reports will cover various aspects of the total processing system.

R. P. Onorati, Captain, USN
Commanding Officer, NORDA

Executive summary

For hydrographic surveys conducted by the Naval Oceanographic Office, the position of the sounding vessel is determined by applying the method of adjustment by variation of parameters. Three types of navigational aids are used: ranging, azimuthal, and hyperbolic. Given data from any combination of at least two navigational aids, a fix may be obtained using an iterative method, which applies successive adjustments to an approximate location and forces it to converge to the most probable position. The magnitude and direction of each adjustment is determined from a least squares solution that minimizes the residual differences between actual navigational observations and imaginary observations calculated as if the ship were at the approximate location.

Acknowledgments

The overall project, Hydrographic Information Handling (HIHAN) System for Automated Smooth Sheet Production, is sponsored by DMA under program element 63701B, Mr. M. Harris, program manager. The DMA program manager is LCDR J. Brodie, and Mr. G. Dupont, Mr. K. Meyer, and Mr. D. Stephenson of the Naval Oceanographic Office are primary contributors toward the total system.

Contents

Sounding vessel position from adjustment by variation of parameters

I. Introduction

The Naval Oceanographic Office conducts bathymetric surveys as part of its continual effort to chart the world ocean. For this work, it is essential to determine sounding positions with a precision compatible with the product.

Coastal survey work is done using three types of navigational aids (navaids): range, azimuth, and hyperbolic (Figs. 1–3). Here, a hyperbolic navaid, which is really a pair of ranging stations, is considered to be a single navaid. From each navaid is obtained a line of position; the intersection of lines of position determine the ship position. If there are more than two lines of position, the intersection generally is not a point. In this event, the most probable position is found by weighting each navaid according to its precision and selecting the position accordingly.

The method presented here differs from other published work in that combinations of different navaid types are used to obtain a fix.

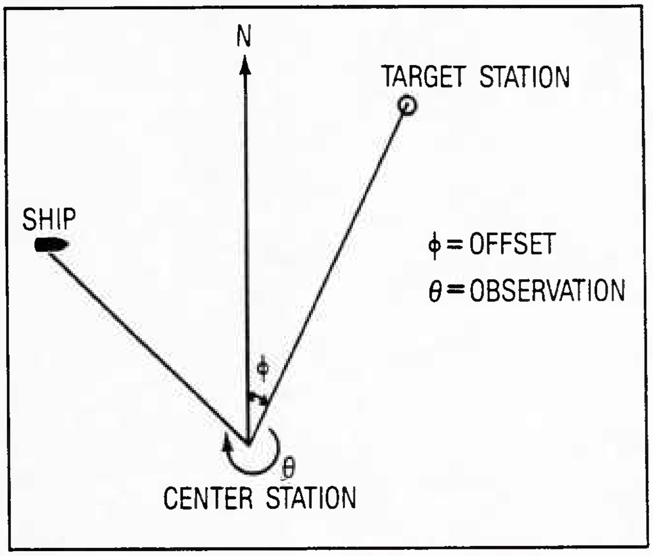

Figure 1. Azimuth navaid. Transit or theodelite is used for this measurement. Azimuth is the sum of the offset and observation angles; this sum yields the bearing of the ship with respect to the center station. The line of position is a straight line.

II. Adjustment by variation of parameters

Given data from any combination of two or more navaids, the most probable position may be calculated by an iterative method. First, the approximate location is specified. For this position a set of imaginary observations is calculated. A new approximate position, which has imaginary observations closer to the actual observations, is then calculated. The process of adjusting an approximate position is repeated until the magnitude of adjustment becomes sufficiently small, at which time the approximate position is taken as the actual position.

The terminology encountered in the literature is somewhat confusing. It is important to distinguish between an actual observation, which is a measurement from a navaid, and an imaginary observation, which is a value that one would expect to measure at a given location. Imaginary observations are often referred to as "computed observations" because the source of the "observations" is a calculation that uses geodetic formulas to compute a given range, range difference, or azimuth.

A. The general model of variation of parameters

The derivation of the general model for variation of parameters follows. A more thorough discussion of this model may be found in Ewing and Mitchell (1970) and Mikhail and Gracie (1981). Initially there are two data items: the approximate position (x, y) and the actual navaid observations (l_i). The first step is to calculate the imaginary observations (I_{ij}) using geodetic formulas (e.g., the

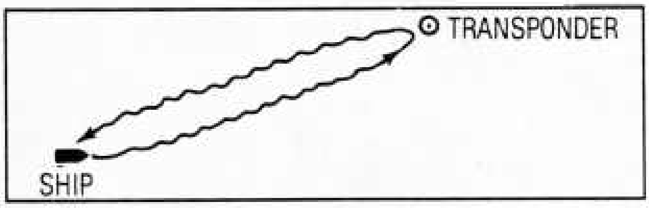

Figure 2. Range navaid. Ship emits pulse that is simultaneously received and retransmitted by a transponder. Round trip distance is proportional to the elapsed time between transmission and reception by the ship. The line of position is a circle.

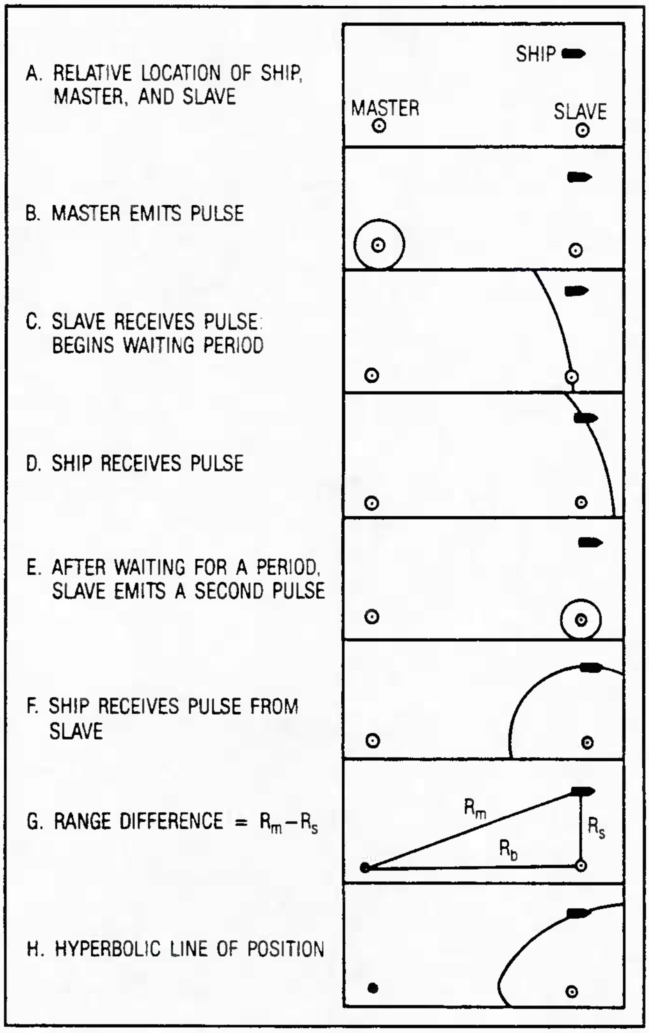

Figure 3. Hyperbolic navaid. Master sends out pulse, which is received by the ship and slave. After waiting for a specified period (the delay), the slave emits a new pulse. The ship measures the difference in time of receipt of master and slave pulses. Knowing the baseline distance (this is published information) the navigator can calculate the difference in distance of the ship from the master and slave. The line of position is a hyperbola, the locus of points having the same difference in distance between the master and slave. Actually the above is an oversimplification of the method used. The signals of the master and slave, rather than being pulses, are continuous sine waves. The ship detects the phase difference between the master and slave; lane count is kept by the instrument and must be initialized at a known location.

Sodano inverse method; Campbell, 1964). Imaginary observations depend on the approximate location.

$$I_{ij} = F_i\,(x,\,y) \tag{1}$$

The subscripts i and j are navaid number and iteration number, respectively.

The goal is to calculate the appropriate adjustment that will "move" the approximate location in such a way that the difference between the actual and imaginary observation is reduced. After adjustment, a new set of imaginary observations ($I_{i,j+1}$) may be calculated. The quantity $I_{i,j+1}$ may be expressed in terms of the previous set of imaginary observations (I_{ij}) and a small change in these values (dI_{ij}),

$$I_{i,j+1} = I_{ij} + dI_{ij} \tag{2}$$

or in terms of the actual observations (l_i) and the deviation from actual observations (v_i).

$$I_{i,j+1} = l_i - v_i \tag{3}$$

The quantity v_i is referred to as the residual difference. Similarly, the difference between actual and imaginary observations is the misclosure (f_i).

$$f_i = l_i - I_{ij} \tag{4}$$

Rearrange Equation (3) to yield

$$v_i = l_i - I_{i,j+1}\ . \tag{5}$$

Now, substitute values of $I_{i,j+1}$ and l_i from Equations (2) and (4)

$$v_i = (f_i + I_{ij}) - (I_{ij} + dI_{ij})$$

$$v_i = f_i - dI_{ij}\ . \tag{6}$$

Expand the dI_{ij} term:

$$dI_{ij} = \frac{\partial F_i\,(x,\,y)}{\partial x}\,dx + \frac{\partial F_i\,(x,\,y)}{\partial y}\,dy\ . \tag{7}$$

Substitute this expression for dI_{ij} into Equation (6).

$$v_i = f_i - \frac{\partial F_i}{\partial x}\,dx - \frac{\partial F_i}{\partial y}\,dy \tag{8}$$

This is the observation equation, and there is one for each navaid. The set of observation equations may be expressed concisely in matrix form as follows.

$$\boldsymbol{v} = \boldsymbol{f} - \boldsymbol{B}\boldsymbol{\Delta}\ , \tag{9}$$

where

$$\Delta = \begin{vmatrix} dx \\ dy \end{vmatrix} \qquad v = \begin{vmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{vmatrix} \qquad f = \begin{vmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{vmatrix}$$

$$B = \begin{vmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} \\ \vdots & \\ \frac{\partial F_n}{\partial x} & \frac{\partial F_n}{\partial y} \end{vmatrix}$$

The next task is to find an expression for the sum of squares of residuals (ϕ). For observations of equal precision,

$$\phi = \sum v_i^2 \, , \tag{10}$$

but this is rarely the case. More generally observations are of unequal precision and are weighted accordingly.

$$\phi = \sum W_i \, v_i^2 \tag{11}$$

In matrix form, this equation is

$$\phi = v^t \; W \; v \tag{12}$$

where

$$W = \begin{vmatrix} W_1 & 0 & \ldots & 0 \\ 0 & W_2 & \ldots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \ldots & W_n \end{vmatrix}$$

Now, substitute the value of v from Equation (9).

$$\begin{aligned} \phi &= (f - B\Delta)^t \; W(f - B\Delta) \\ &= (f^t - \Delta^t B^t) \; W(f - B\Delta) \\ &= (f^t W - \Delta^t B^t W) \; (f - B\Delta) \\ &= f^t Wf - \Delta^t B^t Wf - f^t WB\Delta \\ &\quad + \Delta^t B^t WB\Delta \end{aligned} \tag{13}$$

Since ϕ is a scalar, each term on the right side of Equation (13) is also a scalar. The transpose of a scalar is equal to itself. Therefore, the second term is equal to the third.

$$\Delta^t B^t Wf = (\Delta^t B^t Wf)^t = f^t WB\Delta \tag{14}$$

So Equation (13) reduces to

$$\phi = f^t Wf - 2f^t WB\Delta + \Delta^t B^t WB\Delta \, . \tag{15}$$

To find the appropriate adjustment Δ that yields the minimum value of ϕ, set the partial derivative $\frac{\partial \phi}{\partial \Delta}$ equal to zero and solve for Δ. This is done in Equations (16)–(20). See Mikhail and Gracie (1981, pp. 73 and 322) for calculation of the following derivative.

$$\frac{\partial \phi}{\partial \Delta} = 2f^t WB + 2\Delta^t (B^t WB) = 0 \tag{16}$$

Rearrange and divide by 2.

$$\Delta^t (B^t WB) = f^t WB \tag{17}$$

Again, since each side of the equation is a scalar, each side may be transposed.

$$(B^t WB)^t \Delta = B^t W^t f$$

$$B^t WB\Delta = B^t Wf$$

Now, let $N = B^t WB$ (18)

and $t = B^t Wf$; (19)

then, $N\Delta = t$

and $\Delta = N^{-1} t$. (20)

We have achieved our goal; Equation (20) yields the adjustment to the approximate position. The process of adjustment is repeated until the magnitude of adjustment is sufficiently small.

To solve Equation (20), one must be able to evaluate the B, f, and W matrices. Expressions for these quantities are derived in the next section for ranging, hyperbolic, and azimuthal navaids.

B. Derivation of formulas for the *B, f,* and *W* Matrices

These derivations closely follow the work of Heinzen (1977).

1. B matrix

We start with $F(\phi, \lambda)$ and derive expressions for $\frac{\partial F}{\partial \phi}$ and $\frac{\partial F}{\partial \lambda}$. Here, ϕ and λ are latitude and longitude, respectively, and replace y and x previously used. We will derive expressions for range, hyperbolic, and azimuth navaids.

a. Range

Here $F(\phi, \lambda) = s$, the distance from navaid to ship. For the first part of this derivation, the shape of the earth is approximated by a sphere. The problem is solved by constructing the spherical triangle of Figure 4. (See Ewing and Mitchell, 1970, p. 295-296 for geometrical formulas used here.) The subscript *0* indicates ship; *1* indicates navaid. We will switch back and forth between the two sets of notation used in Figure 4.

Start with the law of cosines.

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\cos s_{01} = \cos(90 - \phi_1)\cos(90-\phi_0)+\sin(90-\phi_1)$$
$$\sin(90-\phi_0)\cos(\lambda_1-\lambda_0)$$

$$\cos s_{01} = \sin \phi_1 \sin \phi_0 + \cos \phi_1 \cos \phi_0$$
$$\cos(\lambda_1 - \lambda_0) \qquad (21)$$

Note that s_{01} is an angle here. Next, differentiate

$$-\sin s_{01}\, ds_{01} = \cos \phi_1 \sin \phi_0\, d\phi_1$$
$$+ \sin \phi_1 \cos \phi_0\, d\phi_0$$
$$- \sin \phi_1 \cos \phi_0 \cos(\lambda_1 - \lambda_0)\, d\phi_1$$
$$- \cos \phi_1 \sin \phi_0 \cos(\lambda_1 - \lambda_0)\, d\phi_0$$
$$- \cos \phi_1 \cos \phi_0 \sin(\lambda_1 - \lambda_0)\, d(\lambda_1 - \lambda_0) \quad (22)$$

In the above equation (ϕ_1, λ_1) is the location of the transponder (or shore transmitter), which is invariant. Therefore,

$$d\phi_1 = d\lambda_1 = 0$$

$$-\sin s_{01}\, ds_{01} = [\sin \phi_1 \cos \phi_0$$
$$- \cos \phi_1 \sin \phi_0 \cos(\lambda_1 - \lambda_0)]\, d\phi_0$$
$$+ \cos \phi_1 \cos \phi_0 \sin(\lambda_1 - \lambda_0)\, d\lambda_0 \ . \qquad (23)$$

The first term on the right, ignoring the $d\phi_0$ factor, is

$$\sin \phi_1 \cos \phi_0 - \cos \phi_1 \sin \phi_0 \cos(\lambda_1 - \lambda_0)\ . \ (24)$$

Using the following identity

$$\cos b \sin c - \sin b \cos c \cos A = \sin a \cos B\ ,$$

Equation (24) can be written as

$$\cos(90 - \phi_1)\sin(90 - \phi_0) - \sin(90 - \phi_1)$$
$$\cos(90 - \phi_0)\cos(\lambda_1 - \lambda_0) = \sin s_{01} \cos \alpha_{01}. \ (25)$$

The second term on the right of Equation (23), again ignoring the $d\lambda_0$ factor, is

$$\cos \phi_1 \cos \phi_0 \sin(\lambda_1 - \lambda_0)$$
$$= \sin(90 - \phi_1)\sin(90 - \phi_0)\sin(\lambda_1 - \lambda_0)$$
$$= \sin b \sin c \sin A. \ (26)$$

From the law of sines we know

$$\frac{\sin a}{\sin A} = \frac{\sin c}{\sin C} \quad \text{and} \quad \sin c \sin A = \sin a \sin C\ .$$

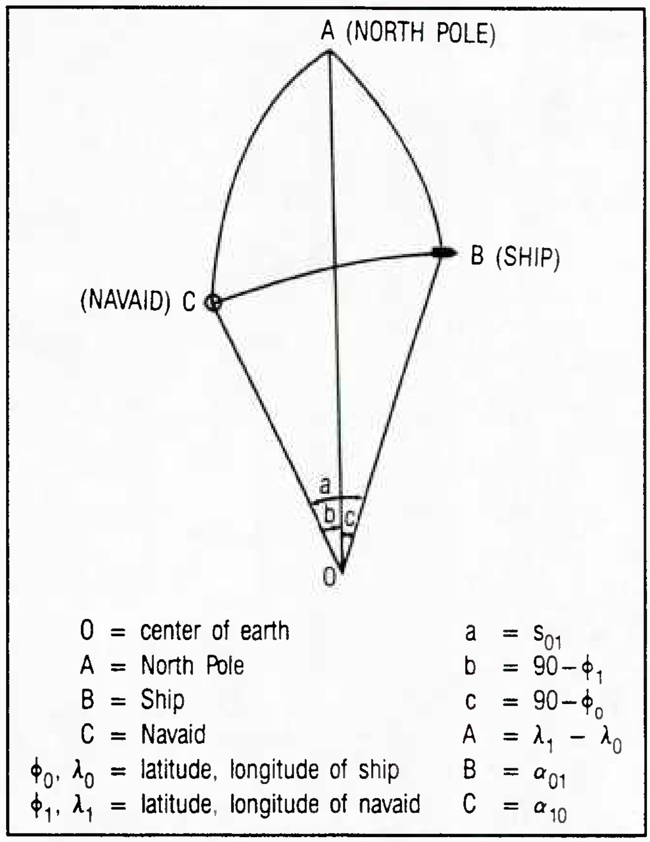

*Figure 4. Spherical triangle used in derivation of **B** matrix expressions. Note that arcs $\widehat{AB}$ and $\widehat{AC}$ are meridians; therefore, $b = 90-\phi_1$, $c = 90-\phi_0$, and $A = \lambda_1-\lambda_0$.*

Therefore, Equation (26) may be written as

$$\begin{aligned} \sin b \sin c \sin A &= \sin b \sin a \sin C \\ &= \sin(90 - \phi_1) \sin s_{01} \sin \alpha_{10} \\ &= \cos \phi_1 \sin s_{01} \sin \alpha_{10} \quad (27) \end{aligned}$$

Substituting Equations (25) and (27) into Equation (23) yields

$$\begin{aligned} -\sin s_{01}\, ds_{01} = \sin s_{01} \cos \alpha_{01}\, d\phi_0 \\ + \cos \phi_1 \sin s_{01} \sin \alpha_{10}\, d\lambda_0 \end{aligned}$$

$$ds_{01} = -\cos \alpha_{01}\, d\phi_0 - \cos \phi_1 \sin \alpha_{10}\, d\lambda_0 \quad (28)$$

To convert the angle s_{01} into distance (arc length), multiply by radius of the earth. We drop the subscripts from s here, and shift from the sphere to the spheroid as our model. Accordingly, the radius in the north-south dimension is the mean radius of curvature (ρ_0) in the plane of the meridian; in the east-west dimension it is the radius of curvature in the prime vertical (ν_0).

$$ds = -\rho_0 \cos \alpha_{01}\, d\phi_0 - \nu_0 \cos \phi_1 \sin \alpha_{10}\, d\lambda_0 \quad (29)$$

Therefore,

$$\frac{\partial s}{\partial \phi_0} = -\rho_0 \cos \alpha_{01}\ , \quad (30a)$$

$$\frac{\partial s}{\partial \lambda_0} = -\nu_0 \cos\phi_1 \sin\alpha_{10}\ . \quad (30b)$$

The line connecting the ship and the navaid is a geodesic line (Fig. 5). Therefore, the azimuth α_{01} (navaid observed from ship) and back azimuth α_{10} (ship observed from navaid) are related as follows.

$$\cos \phi_0 \sin \alpha_{01} = \cos \phi_1 (-\sin \alpha_{10})$$

Now, Equation (30b) may be expressed as

$$\frac{\partial s}{\partial \lambda_0} = \nu_0 \cos \phi_0 \sin \alpha_{01}\ . \quad (31)$$

Finally, Equations (30a) and (31) are rewritten for azimuth α_{01} to be expressed as bearing A_{01} clockwise from N.

$$\begin{aligned} \frac{\partial s}{\partial \phi_0} &= -\rho_0 \cos \alpha_{01} = -\rho_0 \cos(-A_{01}) = \\ &= \rho_0 \cos A_{01} \quad (32a) \end{aligned}$$

$$\frac{\partial s}{\partial \lambda_0} = \nu_0 \cos \phi_0 \sin \alpha_{01} = \nu_0 \cos \phi_0 \sin A_{01} \quad (32b)$$

where s = distance between navaid and ship,
ρ_0 = radius of curvature in the plane of the meridian,
ν_0 = radius of curvature in the plane of the prime vertical,
A_{01} = azimuth; bearing of navaid as observed from ship,
ϕ_0 = latitude of ship, and
λ_0 = longitude of ship.

b. Hyperbolic

Here $F(\phi, \lambda) = d = R_m - R_s$, the difference in range between master and slave. The derivation is analogous to the previous one, yielding

$$\frac{\partial d}{\partial \phi_0} = \rho_0(\cos A_{01m} - \cos A_{01s}) \quad (33a)$$

$$\frac{\partial d}{\partial \lambda_0} = \nu_0 \cos \phi_0(\sin A_{01m} - \sin A_{01s}) \quad (33b)$$

where the subscripts m and s stand for master and slave.

c. Azimuth

Here, $F(\phi, \lambda) = A$ = bearing of ship as observed from navaid station. This derivation is in plane coordinates, which suffices because azimuth measurement may be done

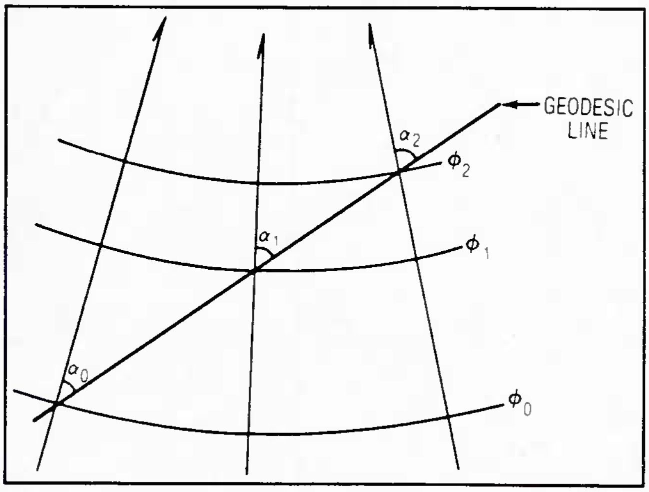

Figure 5. The geodesic line has the following property: $\cos \phi_0 \sin \alpha_0 = \cos \phi_1 \sin \alpha_1 = \cos \phi_2 \sin \alpha_2$. *(After Ewing and Mitchell, 1970, Fig. 4-6.)*

only within a short distance of the ship (the line of sight). Refer to Figure 6.

$$s = (x^2 + y^2)^{1/2} = \text{distance from navaid to ship}$$

$$A = arc\ tan \left(\frac{x}{y}\right)$$

$$dA = \frac{dU}{1 + U^2} \text{ where } U = \frac{x}{y} \text{ and } dU = \frac{dx}{y} - \frac{x}{y^2}dy$$

$$= \frac{1}{1 + x^2/y^2}\left(\frac{dx}{y} - \frac{x}{y^2}dy\right) = \frac{ydx}{y^2+x^2} - \frac{xdy}{y^2+x^2}$$

$$= \frac{1}{s^2}(ydx - xdy)$$

$$= \frac{1}{s}(cos\ A\ dx - sin\ A\ dy) \qquad (34)$$

Now, convert from plane to spherical geometry.

$$dx = \text{east-west distance}$$
$$= r\ cos\ \phi_0\ d\lambda = \nu_0\ cos\ \phi_0\ d\lambda \qquad (35a)$$

$$dy = \text{north-south distance}$$
$$= r\ d\phi = \rho_0\ d\phi \qquad (35b)$$

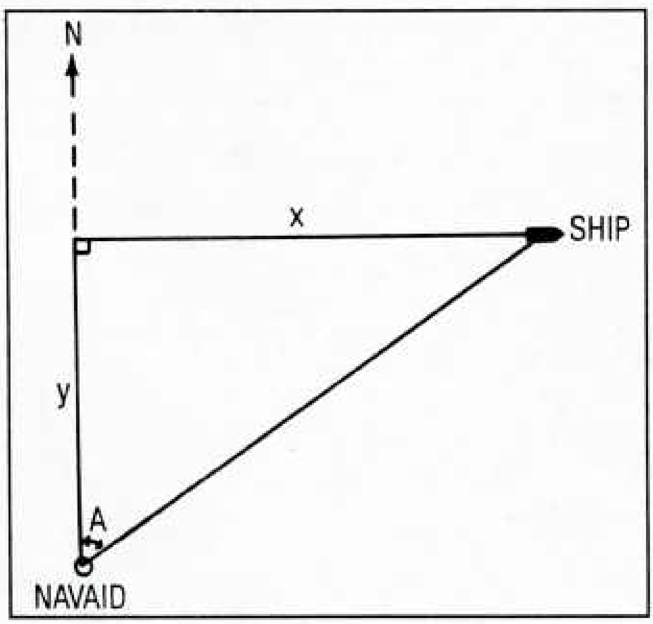

*Figure 6. Plane triangle used for derivation of **B** matrix expression for the case of an azimuth navaid.*

Next, substitute Equations (35a) and (35b) into Equation (34) and calculate the following partials.

$$\frac{\partial A}{\partial \phi} = -\rho_0 \frac{sin\ A}{s} \qquad (36a)$$

$$\frac{\partial A}{\partial \lambda} = \frac{\nu_0}{s} cos\phi_0\ cos\ A \qquad (36b)$$

2. Weight matrix

Expressions for computing weight are empirically derived. The basic principles are simple. Azimuth weights are constants and depend solely on the precision of the instrument. Range weight depends on instrument calibration and the distance to the transponder (precision decreases with distance from the transponder).

Generally, weight is derived from the following equation.

$$W = \frac{\sigma_s^2}{\sigma^2} = \frac{\text{standard variance}}{\text{variance of this type of navaid}}$$

An appropriate value for standard variance has been determined to be 16 m^2, which corresponds to a standard deviation of 4 m. As mentioned, navaid variance for an azimuth instrument is constant (e.g., σ = 0.01° for the Coast Artillery Azimuth Instrument). For ranging instruments, variance is determined from this expression.

$$\sigma^2 = \sigma_0^2 + \left(\frac{s}{10\ km}\right)^2 m^2$$

where σ_0 = standard deviation of the instrument,
s = distance from navaid to ship,
m = meters.

The first term is variance attributable to the precision of the instrument. The second term accounts for that portion of variance that is proportional to the square of the distance.

Hyperbolic weight is treated in a similar fashion to range; the difference is that the calibration errors of the two component ranges cancel each other.

$$\sigma^2 = 2\sigma_0^2 + \left(\frac{d}{10\ km}\right)^2 m^2$$

where σ_0 = (0.02 μsec) (299.67 m/μsec)
d = *range difference*
km = kilometers.

Note that the units for weights are different for azimuth $\left(\frac{m^2}{radian^2}\right)$ and range $\left(\frac{m^2}{m^2}\right)$. This is appropriate because weights are multiplied by residuals, yielding the result ϕ in meters^2.

3. Misclosure

Misclosure is the difference between actual and imaginary observations, and is calculated from the following expressions.

a. Azimuth

$$f = \alpha_{im} - \alpha_{obs} = A_{im} - (A_{offset} + A_{obs}),$$

where the subscript *im* = imaginary and *obs* = observed; A = bearing from north.

b. Range

$$f = s_{im} - s_{obs} = s_{im} - \frac{N\lambda}{2},$$

where N = number of lanes and λ = lane width.

c. Hyperbolic (see Fig. 3g).

$$f = d_{im} - d_{obs}$$

where d_{obs} = C (Δtime − delay)
C = speed of light
d_{im} = imaginary observation of range difference $R_m - (R_s + R_b)$
R_b = baseline distance

Baseline distance is included in these expressions of range difference; it cancels out when d_{obs} is subtracted from d_{im}.

III. Discussion

This method can be conveniently implemented using a computer program. An algorithm is described in Appendix I.

The user must supply an approximate position for each fix. This approximate position is subject to two constraints: it should be close to the actual position (within approximately 100 km); and if either a hyperbolic navaid or a pair of ranging navaids is used, the approximate point must be on the correct side of the baseline connecting the pair of stations.

For a series of fixes from a small survey area, an initial position may be specified, and subsequent fixes can use the preceding fix as their approximate point. This method will work fine until a baseline is crossed, at which point an error will be made. To prevent this error, a new approximate location must be specified on the correct side of the baseline. In most cases, this is taken care of automatically if the approximate point is extrapolated from the last few locations. Still, when a baseline is crossed, the user should check the results carefully.

IV. Conclusions

The methodology described here has been implemented for one year and has survived an initial period of testing. It is part of the NAVOCEANO Hydrographic Post Time System (HPTS). Test data for initially debugging the programs is presented in Appendix II.

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Subroutine MPP

(Given an approximate position and a set of navaid observations, compute position.)

Read navaid observations and depth.

Begin iterative solution: Repeat until positional error is within tolerance or the number of iterations is excessive.

```
1  Do 2 I = 1, 4 (once for each navaid)
     If aid was not selected, or data is unavailable, set weight to zero, go to 2
       CASE (R, A, H)
         RANGE
           Compute the range and azimuth from the approximate position of the ship to this navaid.
           Compute B, f, and W matrix elements.
         AZIMUTH
           Compute the range and azimuth from the approximate position of the ship to this navaid.
           Compute B matrix elements.
           Compute back azimuth (from center station to approximate position of the ship).
           Compute f and W matrix elements.
         HYPERBOLIC.
           Compute range and azimuth from approximate position of ship to the master and slave
             stations.
           Correct ranges for long-range propagation over salt water.
           Compute B, f, and W matrix elements
         END CASE
2  END DO
     (At this point, all matrices are filled.)
     Solve Equation (20) for position correction (dx, dy).
     Apply corrections to ship position.
     If magnitude of correction is within tolerance, RETURN.
       Else if number of iterations is not excessive, GO TO 1.
       Else print ''NO CONVERGENCE''.
RETURN
```

Appendix II. Test data

A. Range-Azimuth

The following data were used to test combinations of range and azimuth navaids. The correct position is 08°15′18.211″S, 116°57′11.205″E.

1. Range data

Lanewidth = 87 m

Navaid	Latitude	Longitude	Observation (lanes)	σ_o (m)
1	8°14′23.0155″S	116°52′43.710″E	96.11	2
2	8°17′18.3105″S	116°55′17.110″E	58.40	2

2. Azimuth data

Navaid	Station	Latitude	Longitude	Observation	σ_o
1	Center	8°14′23.125″S	116°52′43.937″E	317.370°	0.01°
	Target	8°16′38.080″S	116°54′21.159″E		
2	Center	8°17′18.4515″S	116°55′17.151″E	97.479°	0.01°
	Target	8°16′38.0805″S	116°54′21.159″E		

B. Hyperbolic

The following table presents data for 5 positional fixes.

LORAN A
Clarke 1866 spheroid

	Latitude	Longitude	Coding Delay (μsec)	Speed of Propagation (m/μsec)
Master	N41 14 56.330	W69 58 31.4600		
Slave 1	N35 14 25.9300	W75 31 37.8300	1000.	299.692
Slave 2	N43 27 33.4500	W65 28 16.3300	1000.	299.692

Observations (time differences, μsec)		Computed Positions (two methods)		
TD1	TD2	GPTD UNIVAC 1108 O.S. AA2NAA	WITS	Position Difference (seconds)
4400.00	2800.00	N35 24 03.7116	N35 24 03.7112	.0004
		W64 33 05.4840	W64 33 05.4835	.0005
5800.00	1900.00	N39 56 47.1273	N39 56 47.1270	.0003
		W62 48 00.2974	W62 48 00.2966	.0008
3900.00	3300.00	N35 37 49.0375	N35 37 49.0372	.0003
		W67 54 02.0548	W67 54 02.0544	.0004
6000.00	2800.00	N40 23 02.8754	N40 23 02.8752	.0002
		W66 59 26.9214	W66 59 26.9211	.0003
2400.00	3800.00	N35 26 49.4144	N35 26 49.4137	.0007
		W72 30 20.6275	W72 30 20.6269	.0006

Source: NAVOCEANO Computer Branch (1972). WITS Program Documentation. Naval Oceanographic Office, NSTL, Mississippi.